Lecture 8

Streamlines, streaklines, and pathlines

Streamlines, streaklines, and pathlines are field lines in a fluid flow. They differ only when the flow changes with time, that is when the flow is not steady. Considering a velocity vector field in three-dimensional space in the framework of continuum mechanics, we have that:

- **Streamlines** are a family of curves that are instantaneously tangent to the velocity vector of the flow. These show the direction in which a massless fluid element will travel at any point in time.
- **Streaklines** are the loci of points of all the fluid particles that have passed continuously through a particular spatial point in the past. Dye steadily injected into the fluid at a fixed point extends along a streakline.
- **Pathlines** are the trajectories that individual fluid particles follow. These can be thought of as "recording" the path of a fluid element in the flow over a certain period. The direction the path takes will be determined by the streamlines of the fluid at each moment in time.
- **Timelines** are the lines formed by a set of fluid particles that were marked at a previous instant in time, creating a line or a curve that is displaced in time as the particles move.

By definition, different streamlines at the same instant inflow do not intersect, because a fluid particle cannot have two different velocities at the same point. Similarly, streaklines cannot intersect themselves or other streaklines, because two particles cannot be present at the same location at the same instant of time; unless the origin point of one of the streaklines also belongs to the streakline of the other origin point. However, pathlines are allowed to intersect themselves or other pathlines (except the starting and endpoints of the different pathlines, which need to be distinct).

Streamlines and timelines provide a snapshot of some flowfield characteristics, whereas streaklines and pathlines depend on the full time-history of the flow. However, often sequences of timelines (and streaklines) at different instants—being presented either in a single image or with a video stream—may be used to provide insight in the flow and its history.

If a line, curve, or closed curve is used as a start point for a continuous set of streamlines, the result is a stream surface. In the case of a closed curve in a steady flow, the fluid that is inside a stream surface must remain forever within that same stream surface, because the streamlines are tangent to the flow velocity. A scalar function whose contour lines define the streamlines is known as the **stream function**.

Dye line may refer either to a streakline: dye released gradually from a fixed location during the time; or it may refer to a timeline: a line of dye applied instantaneously at a certain moment in time and observed at a later instant.

Mathematical description

Streamlines

Streamlines are defined by,

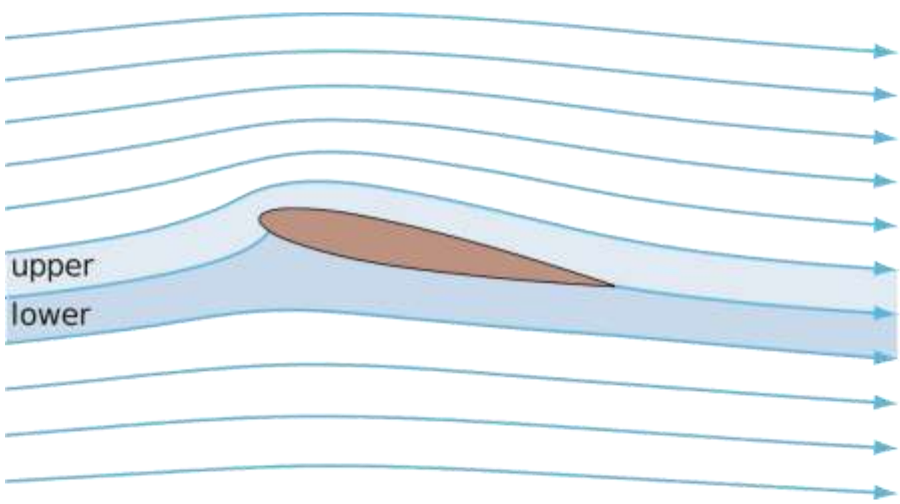
$$\frac{d\vec{x}_S}{ds} \times \vec{u}(\vec{x}_S) = 0,$$

where "X" denotes the vector cross product $\vec{x}_S(s)$ and is the parametric representation of *just one*
streamline at one moment in time.

If the components of the velocity are written $\vec{u} = (u, v, w)$ and those of the streamline as $\vec{x}_S = (x_S, y_S, z_S)$ we deduce

$$\frac{dx_S}{u} = \frac{dy_S}{v} = \frac{dz_S}{w},$$

which shows that the curves are parallel to the velocity vector. Here 's' is a variable which parametrizes the curve $s \mapsto \vec{x}_S(s)$ Streamlines are calculated instantaneously, meaning that at one instance of time they are calculated throughout the fluid from the instantaneous flow velocity field.



Streaklines

Streaklines can be expressed as,

$$\begin{cases} \dfrac{d\vec{x}_{str}}{dt} = \vec{u}_P(\vec{x}_{str}, t) \\ \vec{x}_{str}(t = \tau_P) = \vec{x}_{P0} \end{cases}$$

where, $\vec{u}_P(\vec{x}, t)$ is the velocity of a particle P at location $\vec{x}$ and time t.

Example of a streakline used to visualize the flow around a car inside a wind tunnel.